Exam

تالتاً معادلة برنولي كوول دالي خطية

تذكر ان "خطية"

$$\frac{dy}{dx} + P(x)\,y = Q_{(x)}$$

$$\text{عامل التكامل } \mu = e^{\int P(x)\,dx}$$

$$y = \frac{1}{\mu}\left[\int \mu\, Q_{(x)}\, dx + c\right]$$

$$\frac{dy}{dx} + P(x)\,y = Q_{(x)}\,y^{n}$$

let: $z = y^{1-n}$

$$\therefore\ z' + (1-n)P(x)\,z = (1-n)\,Q_{(x)}$$

$$\text{عامل التكامل } \mu = e^{\int P(x)\,dx}$$

$$z = \frac{1}{\mu}\left[\int \mu\, Q_{(x)}\, dx + c\right]$$

$$y = z^{\frac{1}{1-n}}$$

Ex(1)

$$y' + P(x)\,y = Q_{(x)}\,y^{n}$$

$$y' + 2x\,y = -x\,y^{4}$$

Sol

let: $z = y^{-3}$

$$\therefore\ z' + (1-n)P(x)\,z = (1-n)\,Q_{(x)}$$

$$z' + (-3)(2x)\,z = -3(-x)$$

$$\therefore\ z' - \underbrace{6x}_{P(x)}\,z = \underbrace{3x}_{Q(x)}$$

$$P(x) = -6x$$
$$Q(x) = 3x$$

$$\mu_{\text{عامل التكامل}} = e^{\int P(x)\,dx} = e^{\int -6x\,dx}$$

$$= e^{-6x^2/2} = e^{-3x^2}$$

$$\therefore z = \frac{1}{\mu}\left[\int \mu Q(x)\,dx + c\right]$$

$$\therefore z = e^{3x^2}\left[\int 3x\,e^{-3x^2}\,dx + c\right]$$

$$= e^{3x^2}\left[-\frac{1}{2}\int -6x\,e^{-3x^2}\,dx + c\right]$$

$$= e^{3x^2}\left[-\frac{1}{2}\,e^{-3x^2} + c\right]$$

$$= \cancel{e^{3x^2}} \cdot \frac{-1}{2\cancel{e^{3x^2}}} + e^{3x^2}c$$

$$= e^{3x^2}c - \frac{1}{2}$$

$$\therefore y = z^{\frac{1}{1-n}} = z^{\frac{-1}{3}}$$

$$\therefore y = \left[e^{3x^2}c - \frac{1}{2}\right]^{1/3} \quad \#$$

Ex(2):

$$\overset{P(x)}{y'-y}=\overset{Q(x)}{x}y^5$$

Sol

let $z = y^{-4}$

$\therefore z' + (1-n)P(x)\,z = (1-n)\,Q(x)$

$\therefore z' + 4\,z = -4x$

$\therefore \mu = e^{\int P(x)dx} = e^{\int 4\,dx}$

$= e^{4x}$

$$\boxed{\begin{aligned}P(x) &= 4\\ Q(x) &= -4x\end{aligned}}$$

$\therefore z = \frac{1}{\mu}\left[\int \mu\, Q(x)\, dx + c\right]$

$\therefore z = e^{-4x}\left[\int -4xe^{4x}\, dx + c\right]$

$\int u\,dv = uv - \int v\,du$

let $u = 4x \Rightarrow du = 4$

let $dv = e^{4x} \Rightarrow v = \frac{e^{4x}}{4} \leftarrow F'(x)$

$\therefore -\int 4xe^{4x}dx = 4x\frac{e^{4x}}{4} - \int \frac{e^{4x}}{4}.4dx$

$= xe^{4x} - \frac{e^{4x}}{4} \leftarrow F'(x)$

$\therefore \int 4xe^{4x}\,dx = -xe^{4x} + \frac{e^{4x}}{4} + c$

$$\therefore Z = e^{-4x}\left[-xe^{4x} + \frac{e^{4x}}{4} + c\right]$$

$$= \left(\frac{1}{\cancel{e^{4x}}} \cdot -x\cancel{e^{4x}}\right) + \left(\frac{1}{\cancel{e^{4x}}} \cdot \frac{\cancel{e^{4x}}}{4}\right) + \left(\frac{c}{e^{4x}}\right)$$

$$= -x + \frac{1}{4} + \frac{c}{e^{4x}}$$

$$\therefore y = Z^{\frac{1}{1-n}} = Z^{\frac{-1}{4}}$$

$$\therefore y = \left(-x + \frac{1}{4} + \frac{c}{e^{4x}}\right)^{-1/4}$$

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Ex(3)

$$y' + \frac{2}{x}y = \frac{1}{x^2 y}, \quad x > 0$$

$(x^2 y)$

نظام برنولي

Sol

$$y' + \underbrace{\frac{2}{x}}_{P(x)} y = \underbrace{\left(\frac{1}{x^2}\right)}_{Q(x)} (y)^{-1}$$

$$\text{let}: Z = y^{1-n} = y^2$$

$$\therefore Z' + (1-n)P(x)Z = (1-n)Q(x)$$

$$Z' + \frac{4}{x} Z = \frac{2}{x^2}$$

$$P(x) = 4/x, \quad Q(x) = 2/x^2$$

عامل التكامل

$$\therefore \mu = e^{\int P(x)\,dx} = e^{\int \frac{4}{x}\,dx} = e^{4\int \frac{1}{x}\,dx}$$

$$\boxed{e^{n \ln F(x)} = F(x)^n}$$

$$= e^{4\ln x} = \underline{\underline{x^4}}$$

$$\therefore Z = \frac{1}{\mu}\left[\int \mu\, Q_{(x)}\,dx + c\right]$$

$$\therefore Z = \frac{1}{x^4}\left[\int x^{\cancel{4}\,2}\left(\frac{2}{\cancel{x^2}}\right)dx + c\right]$$

$$= \frac{1}{x^4}\left[\int 2x^2\,dx + c\right]$$

$$= \frac{1}{x^4}\left[\frac{2x^3}{3} + c\right]$$

$$= \frac{2}{3}x + \frac{c}{x^4}$$

$$\therefore y = Z^{\frac{1}{1-n}} = Z^{\frac{1}{2}}$$

$$\therefore y = \left(\frac{2}{3}x + \frac{c}{x^4}\right)^{\frac{1}{2}}$$

$$\text{or} = \sqrt{\frac{2}{3}x + \frac{c}{x^4}} \quad \#$$