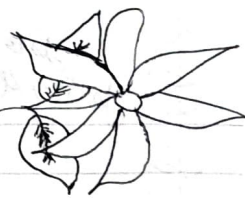


Exam

تالیف معارف برنوس کول والی حیات



$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

نذر "O" ہے

let: $z = y^{1-n}$

$$z' + (1-n)P(x)z = (1-n)Q(x)$$

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$\int \mu = e^{\int P(x) dx}$$

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$$y = \frac{1}{\mu} [\int \mu Q(x) dx + C]$$

$$z = \frac{1}{\mu} [\int \mu Q(x) dx + C]$$

$$y = z^{\frac{1}{1-n}}$$

Ex(1)

$$y' + 2xy = -xy^4$$

sum

let: $z = y^{-3}$

$$z' + (1-n)P(x)z = (1-n)Q(x)$$

$$z' + 3(2x)z = -3(-x)$$

$$z' - 6xz = 3x$$

$P(x) = -6x$
 $Q(x) = 3x$

(21)

$$\int_{0.15}^{0.16} \mu = e^{\int p(x) dx} = e^{\int -6x dx}$$

$$= e^{-6x^2/2} = e^{-3x^2}$$

$$\text{so } Z = \frac{1}{\mu} [\int \mu p(x) dx + c]$$

$$\text{so } Z = e^{3x^2} [\int 3x e^{-3x^2} dx + c]$$

$$= e^{3x^2} \left[-\frac{1}{2} e^{-3x^2} + c \right]$$

$$= e^{3x^2} \left[-\frac{1}{2} e^{-3x^2} + c \right]$$

$$= e^{3x^2} \cdot \frac{-1}{2e^{3x^2}} + e^{3x^2} c$$

$$= e^{3x^2} c - \frac{1}{2}$$

$$\text{so } y = Z^{\frac{1}{1-n}} = Z^{-\frac{2}{3}}$$

$$\text{so } y = \left[e^{3x^2} c - \frac{1}{2} \right]^{-\frac{1}{3}} \neq$$

$$\cancel{E x(2)} \quad y' - y = x y^5$$

su

let $z = y^{-4}$

$$z' + (1-n) p(x) z = (1-n) q(x)$$

$$z' + 4 z = -4x$$

$$\mu = e^{\int p(x) dx} = e^{\int 4 dx} = e^{4x}$$

$$\begin{cases} p(x) = 4 \\ q(x) = -4x \end{cases}$$

$$z = \frac{1}{\mu} \left[\int \mu q(x) dx + C \right]$$

$$z = e^{-4x} \left[\int -4x e^{4x} dx + C \right]$$

$$\int u dv = uv - \int v du$$

$$\text{let } u = 4x \Rightarrow du = 4$$

$$\text{let } dv = e^{4x} \Rightarrow v = \frac{e^{4x}}{4} \leftarrow f'(x)$$

$$- \int 4x e^{4x} dx = 4x e^{\frac{4x}{4}} - \int \frac{e^{4x}}{4} \cdot 4 dx$$

$$= x e^{4x} - \frac{e^{4x}}{4} \leftarrow f'(x)$$

$$\int 4x e^{4x} dx = -x e^{4x} + \frac{e^{4x}}{4} + C$$

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$$\circ \circ z = e^{-4x} \left[-x e^{4x} + \frac{e^{4x}}{4} + c \right]$$

$$= \left(\frac{1}{e^{4x}} \cdot -x e^{4x} \right) + \left(\frac{1}{e^{4x}} \cdot \frac{e^{4x}}{4} \right) + \left(\frac{c}{e^{4x}} \right)$$

$$= -x + \frac{1}{4} + \frac{c}{e^{4x}}$$

$$\circ \circ y = z^{\frac{1}{1-n}} = z^{-\frac{1}{4}}$$

$$\circ \circ y = \left(-x + \frac{1}{4} + \frac{c}{e^{4x}} \right)^{-\frac{1}{4}}$$

$$\text{Ex(3)}$$

$$y' + \frac{2}{x} y = \frac{1}{x^2 y}, \quad x > 0$$

substitution

$$y' + \frac{2}{x} y = \left(\frac{1}{x^2} \right) (y)^{-1}$$

$$\text{let } z = y^{1-n} = y^2$$

$$\circ \circ z' + (1-n) P(x) z = (1-n) Q(x)$$

$$z' + \frac{4}{x} z = \frac{2}{x^2}$$

$$P(x) = \frac{4}{x}, \quad Q(x) = \frac{2}{x^2}$$

(24)

$$\int_0^1 \frac{1}{x} dx = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

$$e^{\ln f(x)} = f(x) \quad 4 \ln x = x^4$$

$$\therefore Z = \frac{1}{4} \left[\int 4 \ln x dx + C \right]$$

$$\therefore Z = \frac{1}{x^4} \left[\int x^{\frac{2}{4}} \left(\frac{2}{x^2} \right) dx + C \right]$$

$$= \frac{1}{x^4} \left[\int 2x^2 dx + C \right]$$

$$= \frac{1}{x^4} \left[\frac{2x^3}{3} + C \right]$$

$$= \frac{2}{3}x + \frac{C}{x^4}$$

$$\therefore y = Z^{\frac{1}{1-n}} = Z^{\frac{1}{2}}$$

$$\therefore y = \left(\frac{2}{3}x + \frac{C}{x^4} \right)^{\frac{1}{2}}$$

$$\text{or } = \sqrt{\frac{2}{3}x + \frac{C}{x^4}}$$

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