$f(x) \neq 0$ y_p ثانياً : نحولها طبقاً لـ "أويلر"

لحل نفس طريقة حل البارامتر

بنفس الخواص مش هنزود حاجة غير حاجة بسيطة وهي تغيير $f(x)$

3 حالات

$$y_G = y_c + y_p$$

Note

$$\boxed{f(x),\ \frac{1}{x^2}}$$

$$y_c = c_1 y_1 + c_2 y_2$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$\boxed{u_1 = \int \frac{w_1}{w}\, dx} \qquad \boxed{u_2 = \int \frac{w_2}{w}\, dx}$$

(1) $w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = (y_1 y_2') - (y_2 y_1')$

(2) $w_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix} = (0 \times y_2') - (y_2 f(x))$

مش بتاعت رأس السؤال

(3) $w_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} = (y_1 f(x)) - (0 \times y_1')$

$\therefore y_p = u_1 y_1 + u_2 y_2$

$y_p = \int \frac{w_1}{w} dx\, y_1 + \int \frac{w_2}{w} dx\, y_2$ #

Ex(2)

فقطها وضبرا x^2 — $x^2y'' - 2xy' - 4y = \frac{1}{x}$ — عالبا $\times \frac{1}{x^2}$

sln

$$y'' - \frac{2}{x}y' - \frac{4}{x^2}y = \frac{1}{x^3}$$

$\therefore y_G = y_c + y_p$

$r^2 - 3r - 4 = 0$

$(r-4)(r+1) = 0$

$r_1 = 4,\ r_2 = -1$

$\therefore y_c = c_1 x^{r_1} + c_2 x^{r_2}$

$\boxed{y_c = c_1 x^4 + c_2 x^{-1}}$

let

$y_p = u_1 \overset{y_1}{x^4} + u_2 \overset{y_2}{x^{-1}}$

$$y_p = x^4 \int \frac{w_1}{w}dx + x^{-1}\int \frac{w_2}{w}dx$$

① $w = \begin{vmatrix} x^4 & x^{-1} \\ 4x^3 & -\frac{1}{x^2} \end{vmatrix} = \left(x^4 \cdot \frac{-1}{x^2}\right) - \left(\frac{1}{x}\cdot 4x^3\right)$

$$w = -x^2 - 4x^2 = -5x^2$$

② $w_1 = \begin{vmatrix} 0 & x^{-1} \\ \frac{1}{x^3} & -\frac{1}{x^2} \end{vmatrix} = \frac{-1}{x}\cdot\frac{1}{x^3} = \frac{-1}{x^4}$

③ $w_2 = \begin{vmatrix} x^4 & 0 \\ 4x^3 & \frac{1}{x^3} \end{vmatrix} = x^4 \cdot \frac{1}{x^3} = x$

$$\therefore y_p = x^4\int \frac{w_1}{w}dx + x^{-1}\int\frac{w_2}{w}dx$$

$$= x^4 \int \frac{-x^{-4}}{-5x^2}dx + x^{-1}\int \frac{x}{-5x^2}dx$$

$$= x^4 \frac{1}{5}\int x^{-6}dx + x^{-1}\frac{-1}{5}\int \frac{1}{x}dx$$

$$= x^4 \frac{1}{5}\left(\frac{x^{-5}}{-5}\right) + x^{-1}\frac{-1}{5}(\ln x) = \frac{-1}{25}x^{-1} - \frac{1}{5}x^{-1}\ln x$$

y_p

$$\therefore y_G = y_c + y_p = c_1x^4 + c_2x^{-1} - \frac{1}{25x} - \frac{\ln x}{5x}$$ #